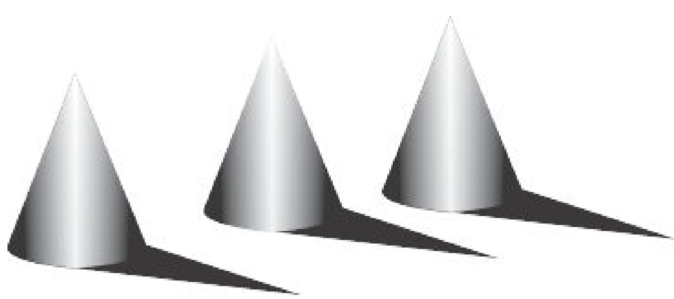
The Research Base of the *Making Sense of Problem Solving* Program in Educational Literature



Teaching Mathematics Through Problem Solving

Fennema, et al (1996) found that some instructional practices were associated with higher student achievement:

- Providing time for students to work with mathematical ideas in problem solving contexts
- Providing opportunities for students to converse with each other about mathematical ideas
- Adapting instruction to the problem-solving level of students

During the five years of the QUASAR project, Silver and Stein (1996) engaged teachers in developing and implementing an instructional style that engaged urban middle school students in developing meaningful understanding of mathematical ideas through problem solving. They had strikingly positive results in student achievement.

Gearhart et al (1999) found that students learn more mathematics in classrooms that:

- Engaged students in problem solving with rich problems
- Assisted students in seeing underlying links among various mathematical concepts and symbols.

Brown (1997) found that when students were actively engaged in figuring out why things work the way they do, they understood core information deeply and were able to transfer it to new situations.

Stein, Smith, Henningsen, Silver (2000), describe four levels of cognitive demand typically used in mathematics instruction: memorization, procedures without connection, procedures with connections, and doing mathematics (problem solving). They cite numerous examples of teachers who could only engage students in working collaboratively, discussing alternative approaches to solving mathematical tasks, and justifying their solutions, if they chose rich problem solving tasks. Tasks with less cognitive demand did not engage students in mathematical discourse. Instead, discussions of less demanding tasks focused on procedures and correct answers.

John Van de Walle (1993) describes three characteristics of effective problem-based tasks:

- Focus students on the mathematical ideas that are embedded in the task
- Be accessible to all students (challenging but within reach)
- Include requirement of justifications and explanations

The *Making Sense of Problem Solving* supplementary curriculum is designed to support the *NCTM Curriculum Focal Points – A Quest for Coherence* and the *NCTM Principles and Standards for School Mathematics.* The *NCTM Principles and Standards* (NCTM, 2000, 3.) begins with A Vision for School Mathematics:

> *Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively*

Targeting the NCTM Curriculum Focal Points:

The Curriculum Focal Points identify core topics of mathematics at each grade level, pre-K–8. They are intended to be organizing structures for curriculum and instruction. "Because the focal points are core structures that lay a conceptual foundation, they can serve to organize content, connecting and bringing coherence to multiple concepts and processes taught at and across grade levels." (National Council of Teachers of Mathematics [NCTM] 2006, 5.)

If the Focal Points can be thought of as a skeletal structure, the Connections are ligaments and tendons. The Connections serve two purposes:

1. *The Connections provide introductory and continuing experiences related to the focal points identified for other grade levels.*

2. *They identify ways in which the grade level's focal points can support learning in relation to strands that are not focal points at that grade level.* (NCTM 2006, 8.)

In his spring 2007 NCTM President's message, Skip Fennel explained the purpose of the Curriculum Focal Points:

> *The purpose of the focal points is to ensure direct attention, or focus, so that what is taught can be covered thoroughly and understood deeply, with continuous engagement in problem solving, reasoning and proof, communication, connections, and related representations.*

Theories and Findings Used in the Development of *Making Sense of Problem Solving*

Fennema et al. (1996) reported on a longitudinal study of primary-grade teachers who used Cognitively Guided Instruction (CGI) as a system of research-based models of students' mathematical thinking. Students in the study, in many cases, improved by one standard deviation on achievement tests.

Making Sense of Problem Solving encourages the use of representations that will stimulate processing in multiple brain centers. TTT has used ideas from a number of researchers in developing the instructional suggestions used in our program.

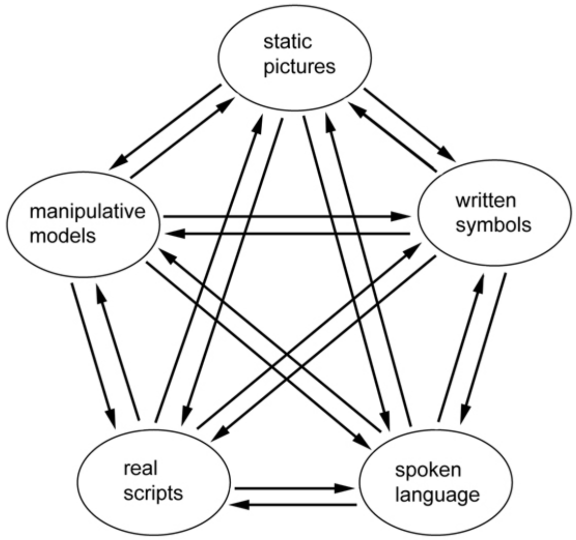
The importance of stimulating the right brain and adding that enhanced functionality to already well-developed left-brain processing is a central thesis in Daniel H. Pink's (2007) *A Whole New Mind: Why Right-Brainers Will Rule the Future.* Pink argues that what was important in the Information Age—left brain sequential, literal, functional, textual and analytic thinking—is no longer sufficient. As we move into the Conceptual Age, there is more need for right-brain simultaneous, metaphorical, aesthetic, contextual and synthetic thinking; the type of thinking that is not assessed well on traditional tests.

Using a variety of representations of mathematical ideas, which would stimulate various parts of the brain, is recommended in Principles and Standards for School Mathematics (NCTM. 2000, p. 67):

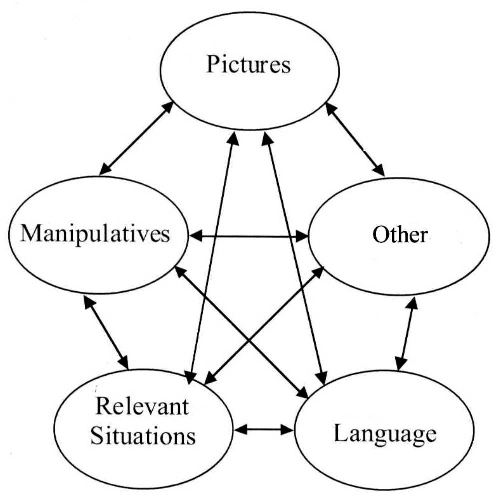
> *Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling.*

Gearhart et al (1999) found that students learn more mathematics in classrooms that assist them in seeing underlying links among various mathematical concepts and symbols.

Lesh, Post and Behr (1987) proposed a model of how mathematical ideas can be representedheir model identifies topics that would stimulate various parts of the brain.



Lisa Clement (2004) proposed adaptations to the Lesh, Post and Behr model in *A Model for Understanding, Using, and Connecting Representations.* The underlying idea of encouraging students to use a variety of representations, which would stimulate various parts of the brain, can be clearly seen in Clement's Mathematical Representations Model:



Children from low-income backgrounds, particularly low-income African-American backgrounds, are just as effective in selecting appropriate strategies for solving problems as any other students. However, they often do not possess adequate factual knowledge. Kerkman and Siegler's work suggests that the reason they did not have adequate factual knowledge was because they had less practice in solving problems. The findings indicate that engaging students in problem solving will be the most useful way to improve their arithmetic skills. (Kerkman & Siegler, 1993)

Children learn best when they are allowed to use the strategy they wish to use. Immature strategies usually become used less often as children develop more advanced strategies. Students who use a variety of strategies tend to learn more effectively than those who do not. (Alibali & Goldin-Meadow, 1993; Chi, de Leeuw, Chiu, & LaVancher, 1994; Siegler, 1995).

Heibert and Wearne (1996) found a strong positive correlation between students' conceptual understanding of multi-digit addition and subtraction and the ability to invent procedures and to reflect on why a correct procedure is correct.

The research conducted by Rittle-Johnson and Alibali (1999) with fifth graders on mathematical equality problems suggests that students should be engaged in conceptual instruction before they are taught procedures.

Many studies have found that well-focused cooperative learning among peers often stimulates problem solving and reasoning to a greater degree than students achieve when working independently (Gauvain & Rogoff, 1989; Teasley, 1995; Webb, 1991).

Siegler (1995) found that analytic thinking can be encouraged, even in kindergarteners, by asking children to explain the correct solutions that other people develop. Stigler & Perry (1990) reported on the benefits of an analytic strategy often used in Japanese classrooms and associated with high achievement in mathematics. Students are asked to explain why incorrect solutions are incorrect. Siegler (2002) found that asking students to both explain why correct solutions were correct and also why incorrect solutions were incorrect led to greater mastery than simply asking why solutions were correct.

Shimizu (2003) contrasted direct instruction methods to an instructional model typically used in Japanese classrooms. Japanese lessons are typically organized around multiple solutions to a single problem, because of the belief that students learn most effectively when they are engaged in solving a challenging problem. They also believe that the lesson highlights appear during the discussion of solutions. Teachers prepare for lessons by anticipating a wide range of student solutions and methods, so that they are prepared to lead a discussion that draws out and highlights the essential mathematical ideas that students are likely to present.

Section I of each grade level *Making Sense of Problem Solving* Teacher's Guide includes lessons that emphasize instructional strategies that are to be used throughout this yearlong supplementary curriculum. Teachers who use our program frequently report these outcomes, which are goals of the *NCTM Principles and Standards for School Mathematics (2000, p. 3):*

1. **Students excited by and interested in their activities.** When math is taught with a problem-solving spirit, and when children are allowed to make their own hands-on mathematical discoveries, math can be engaging for all students.
2. **Students posing and solving meaningful problems.** When students are challenged to use mathematics in meaningful ways, they develop their reasoning and problem-solving skills and come to realize the potential usefulness of mathematics in their lives.
3. **Students working together to learn mathematics.** Students learn mathematics well in cooperative settings where they can share ideas and approaches with their classmates.
4. **Students writing, talking and representing math topics every day.** Putting thoughts into words and/or drawings helps to clarify and solidify thinking.

Design of Professional Development Program

The *Making Sense of Problem Solving Professional Development* program is grounded in sound theories of learning in general, and in theories of adult learning in particular.

Schifter and Fosnot (1993) developed four principles of teacher education:

- Teacher education should be based on the same pedagogical principles as classroom mathematics instruction
- Teachers must become mathematical learners, engaged in solving problems, if they are expected to teach for understanding
- As teachers develop new instructional strategies, regular classroom consultation helps to sustain teachers' learning
- Facilitating collaboration among teachers is essential to the process of reforming instruction

Lappan and Briars (1995) summarized views of student learning that are supported by NCTM:

- How students learn something is intrinsically connected to what they learn.
- Learning occurs best when students:
    - Engage in mathematical discourse and interaction
    - Are actively involved
    - Work individually, in pairs, and in cooperative groups
    - Review, evaluate and revise one another's work

Deborah Shifter's (1998) research in adult learning suggests that teachers in professional development programs for mathematics education need to:

- Have opportunities to learn mathematics in the same ways they are expected to teach it
- Struggle with significant mathematical ideas
- Justify their thinking to their peers
- Investigate alternative solutions that are proposed by peers

Many researchers such as Brown, Smith, & Stein (1996), Campbell & White (1997), Heaton (1994, 2000), Kilpatrick, Hancock, Mewborn & Stallings (1996), and Swafford, Jones & Thornton (1997) have consistently found evidence that teachers need to gain insights into the key conceptual issues of the mathematical topics they will teach, in order to successfully develop their practice.

Thompson and Thompson (1994, 1997) found that besides knowing the mathematics, teachers must listen carefully to student reasoning and have the solid mathematical base to understand student thinking.

Kazemi & Franke (2000) found that the best results of professional development efforts happened when teachers saw their classrooms as extensions of the professional development.

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